SAILBOAT DESIGN BY RESPONSE SURFACE OPTIMIZATION

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Abstract. A methodology is presented by which sailboat designers can explore a given design space, and derive the set of parameters and corresponding geometry of a sailboat hull that achieves the best performance for a given sailing condition. The methodology consists of developing, within a design space and using an advanced modeller, several parametric variations of a baseline design, for each of which a measure of performance is computed. A mathematical relationship between the design parameters and the measure of performance is derived next, which is used with an optimisation solver, to compute the maximum of the measure of performance and obtain the corresponding design parameters and geometry. The case study presented in this paper is for a sailboat hull, but it is equally applicable to the design development of any component for which a measure of performance can be computed and related to its design parameters.

NOMENCLATURE

BWL	Waterline Beam
Cp	Prismatic Coefficient
GM	Metacentric Height
LCB	Longitudinal Centre of Buoyancy
LCF	Longitudinal Centre of Flotation
LPP	Length between Perpendiculars
Tc	Canoe Body Draft
VCG	Vertical Centre of Gravity
WPA	Water Plane Area
WSA	Hull Wetted Surface Area

FRIENDSHIP-Modeler Parameters

AftAreaCoeff	area coefficient of the area beneath the aft part of the deck line
BeamDeck	maximum beam at deck level
Draft	maximum draft of canoe body
FabLength	initial parametric acceleration of the surface at the centre plane edge upwards
FasLength	initial parametric acceleration of the surface at the deck edge downwards
ForAreaCoeff	area coefficient of the area beneath the fore part of the deck line
Lpp	length between perpendiculars
XPosMaxDraft	longitudinal position of the maximum draft
XPosMaxBeam	longitudinal position of the maximum beam

1. INTRODUCTION

Dimensions, shapes and mass properties are attributes that skilful designers have been able to blend intuitively in the development of performance sailboats for many years. As technology progressed and the understanding of the physics involved became clearer, several parameters that characterize the geometry, mass properties, and the forces and moments acting on a sailboat have been derived with the purpose of correlating, in a quantitative manner, sailing performance with these parameters.

At the onset of any typical design project designers are faced with several constraints that limit their ability to design a sailboat with complete freedom. Performance sailboat designers face the challenge of managing opposing requirements such as developing a fast design within the constraints of a handicap or box rule. In addition, limited time, economical and technical resources often reduce the possibility of investigating the performance of alternative designs before the boat is built.

Despite the recent advances in performance prediction modelling, the systematic exploration of the design space with the objective of finding the optimum design for a given set of requirements continues to be a tedious and time consuming effort. More often than not, this effort does not cover the complete range of possible designs. The methodology presented in the paper was developed with the objective of facilitating this process. It consists of :

1. developing, within the design space, several parametric variations of a baseline design,
2. computing, for each design, a measure of performance,
3. developing a mathematical relationship between the design parameters and the measure of performance,
4. computing the maximum of the measure of performance to obtain the corresponding "best" design parameters and geometry.

The methodology is demonstrated developing the hull of a modern 100 foot baseline sailboat for which several hull designs were generated using an advanced parametric modelling system. Within this system, the hull geometry is directly generated from a small set of form parameters, while the characteristics and constraints

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of the hull design are maintained, and the shape is optimized with respect to fairness.

Each parametric variation was characterized from the standpoint of performance by the Velocity Made Good (VMG), which was predicted by a Velocity Prediction Program (VPP) that uses the Delft Systematic Yacht Hull Series (DSYHS) and an International Measuring System (IMS) type model for the hydrodynamic and aerodynamic forces and moments, respectively.

For a given True Wind Speed (TWS) in the upwind condition, VMGs and design parameters were fitted using radial basis functions to provide a continuous mathematical representation of the data (i.e. a response surface) and means to allow for the evaluation of the VMGs anywhere in the design space.

Finally, an optimisation solver was used in conjunction with the response surface to find the set of design parameters, and corresponding geometry, that would produce the maximum VMG.

2. DESCRIPTION OF THE METHODOLOGY

The standard approach to develop the design of performance sailboats is to use VPPs to compute measures of performance. VPPs based on regression formulas (such as the DSYHS), would allow for the quick calculation of VMGs of the relatively large number of design variations that could be produced with a parametric modeller. In these circumstances, finding the combination of hull parameters that produces the maximum VMG could be as easy as sorting the results. However, VPPs used in the development of performance sailboat designs often rely on CFD and/or towing tank experiments data to produce measures of performance of the design variations explored. This approach produces better performance predictions compared to a regression formula approach, but at the expense of examining a lesser number of designs and possibly omitting regions of the design space where the optimum design could exist. The simulation and experimentation work necessary to derive measures of performance is typically very intensive, and in addition, resources are often limited. For these reasons, the number of design variations studied in the design of a performance sailboat is, in general, relatively small.

The basis of the proposed methodology is that a measure of performance, such as VMG, varies as a function of the design parameters that define the design space. Such a function, would exhibit a distinct maximum (possibly global) which could be found with an optimisation solver, provided that the function can be mathematically represented and evaluated. Given the multiple dimensionality of these functions (i.e. as many dimensions as design parameters chosen for the design space), these functions are often referred to as “surfaces” or “response surfaces” due to the input-output relationship they represent. This procedure is referred to as response surface optimisation (RSO).

In the context of this paper, the solution to the problem of parametric development of a performance sailboat design and its optimization is to produce an interpolating multi-dimensional response surface of a measure of performance that would retain the topological features of the design space using relatively few design variations. The maximum could be then obtained from the response surface using an optimisation solver.

In order to develop a solution to this problem three elements are needed. One, a sampling strategy that would produce design variations uniformly distributed over the design space. The second, an algorithm that would interpolate the resulting data and create the response surface. And third, an optimisation solver capable of a finding the global maximum.

In order to provide a random, uniform distribution of hulls within the design space a Sobol sequence [1] was used. Being actually a quasi-random strategy, the Sobol sequence ensures to always have a well represented design space, in a statistical sense, with increasingly finer resolution as more samples are produced, while avoiding clustering the design parameters. An additional advantage of this strategy is that the algorithm avoids “grid lines” and thus, gives a more stable basis for subsequent multi-dimensional interpolation; much more effective than other strategies such as the Monte Carlo. Figure 1 shows a section, LPP versus Tc, of a design space sampled using a Sobol sequence.

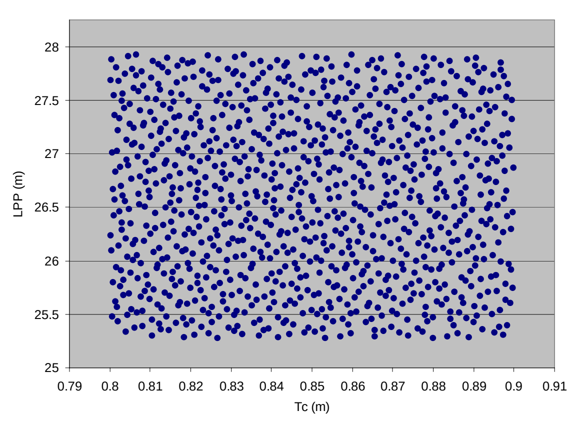

Figure 1. Example of a design space section sampled using a Sobol sequence.

The proposed algorithm to generate the response surface is based on radial basis functions or RBFs. The main strength of RBFs is their ability to elegantly and accurately interpolate scattered data in arbitrary dimensions, without using data on a grid. Several other algorithms could be used such as polynomials, splines, neural networks, but RBFs have proven to be computationally fast and accurate for this application.

For the case study presented in this paper, 946 hull design variations were generated using FRIENDSHIP-Modeler, a fully parametric design tool. Corresponding VMGs, for upwind and a range of TWSs, were computed using FRIENDSHIP-Equilibrium, a VPP based on the DSYHS and IMS type hydrodynamic and aerodynamic models, respectively. The only reason for using this type of VPP was to quickly generate VMGs for the large number of variations generated with the parametric modeller and produce the data needed to demonstrate the RSO methodology. In practice, measures of performance can be computed in any desired manner. Even full CFD modelling could be employed as currently performed for commercial vessels, see for instance [2].

The 946 design variations produced a fine resolution sample of the design space that was used to find, by sorting the resulting VMGs, the "best" set of design parameters, and use those to identify the region of the design space where the "best" design may exist. Furthermore, an additional set of design parameters were obtained by searching for a further maximum with a VPP driven Tangent search optimiser in the vicinity of this point. This produced an additional set of "best" design parameters.

Response surfaces were computed using all 946 points and only 30 points to demonstrate that despite the use of a limited number of design variations, the proposed RSO methodology would yield sets of "best" design parameters that would cluster in a region of the design space where the "best" measure of performance may exist.

The optimisation solver used was a Generalized Reduced Gradient (GRG) solver featuring a clustering technique to allow for the computation of the global maximum.

3. CASE STUDY

3.1 Baseline design

The hull of a modern 100 ft sailboat, featuring a keel with bulb and rudder, was used to demonstrate the methodology. Table 1 summarizes the baseline design principal characteristics:

Displacement (kg)	52,491
LWL (m)	26.652
BWL (m)	5.349
Tc (m)	0.85
WSA (m2)	107.8
WPA (m2)	98.11
LCB (m)	14.78
LCF (m)	15.312
GM (m)	3.593
VCG (m)	-0.694
Cp	0.548
I (m)	39.7
J (m)	11.49
P (m)	38.5
E (m)	12.8

Table 1. Baseline design principal characteristics.

3.2 Development of hull parametric variations

Nine hundred forty six hull designs were generated using FRIENDSHIP-Modeler, a software system for the advanced parametric modelling of yachts and ships, for details see [3]. Within this system, the hull geometry is directly generated from a small set of form parameters, while the characteristics and constraints of the hull configuration are maintained and the shape is optimised with respect to fairness. The modelling approach is based on multiple nested optimisations of uniform B-spline curves defining sectional properties of the hull for each longitudinal position. These curves are build from a flexible selection of properties. For instance, the plan view of the deck can be specified by its forward end at the bow, the position of the maximum beam and aft end at the transom. Tangency and curvature conditions can be optionally applied. The modelling algorithm then arranges the controlling vertices in such a way that the specified properties are met while the curve is kept as fair as possible. Finally, the FRIENDSHIP-Modeler can be easily coupled to a sophisticated Sobol sequence generator to sample the design space, which is defined by design parameter ranges.

The design space selected for this case study, defined by typical hull dimensions and hydrostatic coefficients, is summarized in Table 2. However, the FRIENDSHIP-Modeler generates hull geometries from form parameters. Out of about 60 form parameters, nine of these parameters were selected and varied for this case study, while the rest were kept constant. The form parameters and ranges which correspond to the design space shown in Table 2, are shown in Table 3. The baseline hull and appendages are depicted in Figure 2.

1. all variations had the same volume
2. $4.8 \leq \text{BWL} \leq 5.5$ m
3. $0.80 \text{ m} \leq \text{Tc} \leq 0.90$ m
4. $0.52 \leq \text{Cp} \leq 0.59$
5. $95\% \leq \text{LWL} \leq 105\%$ of baseline LWL
6. $55\% \leq \text{LCB} \leq 57\%$ of baseline LWL
7. $90\% \leq \text{LCF} \leq 110\%$ of baseline LWL

Table 2. Design space.

Parameter	Range
draft	0.80 to 0.90 m
beamDeck	6.22 to 7.08 m
lpp	25.27 to 27.93 m
xPosMaxDraft	12.00 to 16.80 m
xPosMaxBeam	16.30 to 18.30 m
forAreaCoeff	0.61 to 0.66
aftAreaCoeff	0.84 to 0.88
fabLength	0.16 to 0.21
fasLength	0.24 to 0.29

Table 3. FS-Modeler form parameters.

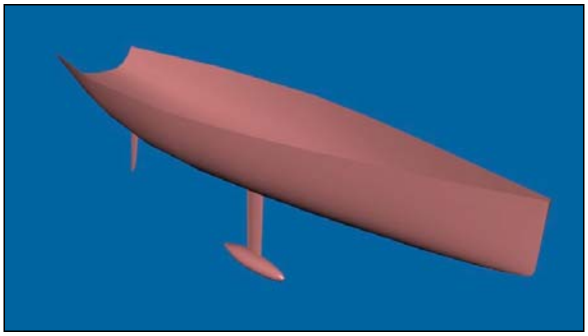

Figure 2. Baseline hull.

3.3 Performance prediction

For each design variation generated with the FRIENDSHIP-Modeler, upwind velocity predictions were performed and VMGs were computed with FRIENDSHIP-Equilibrium, a VPP that uses the DSYHS and an IMS type model for the hydrodynamic and aerodynamic forces and moments, respectively. For details see [4].

Velocity predictions were produced for a range of True Wind Speeds (TWS) of 3 to 11 m/sec (5.83 to 21.38 knots). Since it would not be possible to design a sailboat for optimum performance over the whole range of TWS, designers typically select a TWS range for which the design would be optimised. For this case study, the selected TWS was 13.6 knots (7 m/s).

3.4 Response surface

Several algorithms are commonly used to construct response surfaces. Among the most popular are polynomials, splines, neural networks and radial basis functions. For this case study, the response surface was derived by means of radial basis functions of the Radial-Polynomial type. These were selected because of their computational efficiency, ease of use, and fundamentally, because they work well with multi-dimensional scattered data. The application of radial basis functions on scattered data was performed initially by Hardy [5].

3.5 Optimisation

The optimisation solver used in this study was a Generalized Reduced Gradient solver which, like other gradient-based solvers, is meant to find local solutions. Response surfaces, like the ones derived in this study, will feature irregularities that most likely would lead the solver to find local solutions instead of the desired global solution (maximum). Traditional ways of dealing with this problem includes starting the solver from initial conditions (guesses) derived from the knowledge of the response surfaces topology, or by a trial-and-error method by which the solver is started from a multitude of points (multistart) to see which point led to the best (global) of the local solutions. The former approach is very difficult since prior knowledge of the response surface topology is often unavailable or difficult to determine. The latter approach could be time consuming or, if not organized and conducted in a systematic manner, would lead to the same local solution identified several times, thereby leading to an inefficient global search.

The optimisation solver used in this study included a “multi level single linkage” clustering technique by which initial conditions are randomly sampled and clustered into groups that would likely lead to the same local solution. The solver is run next to find the local solution within each cluster followed by a Bayesian analysis to determine when to stop sampling new clusters. The solution obtained with this method converges with a high degree of probability to the global solution searched.

4. ANALYSIS OF RESULTS

The verification of the methodology was performed in five steps. It was important to verify that:

1. the “best” sets of design parameters that were obtained did indeed produced designs faster than the baseline,
2. VMGs corresponding to RSO derived design parameters were in agreement with those derived using the VPP for the same set of parameters,
3. VMGs corresponding to RSO derived design parameters were in agreement with those derived using the VPP driven Tangent search optimisation method,
4. the VMG corresponding to the RSO derived design parameters using only 30 design variations was in agreement with the VMG where all 946 variations were used, and
5. the “best” sets of design parameters clustered in a region of the design space, permitting the identification of optimum performance.

Table 4 shows baseline and “best” VMGs. VMGs indicated as RSO were obtained according to the response surface optimisation method.

Design		VMG (knots)	Better than Baseline
Baseline, Hull 0		8.07	
Hull 223 maximum VMG of all 946 parametric variations		8.21	1.7%
Tsearch VPP driven Tangent search optimisation in the vicinity of Hull 223		8.30	2.8%
Optimum_946	VPP	8.22	1.8%
	RSO	8.24	2.1%
Hull 22 maximum VMG of 30 parametric variations		8.16	1.1%
Optimum_30	VPP	8.25	2.2%
	RSO	8.26	2.3%

Table 4. Baseline and “Best” VMGs.

Note that in all cases the performance of the baseline design is exceeded. Furthermore, VMGs derived by RSO are essentially the same (less than 0.5% difference) as those computed with the VPP using the corresponding design parameters. The Optimum_946 used all 946 design variations, while Optimum_30 used only 30. VMGs derived by RSO are within 1% of the VMG obtained by means of a Tangent search (Tsearch) optimisation in the vicinity of Hull 223, which was the "best" design in the 946 design variations. Finally, the RSO computed VMG from the 30 design variations is within 0.25% of the VMG computed with all the 946 design variations.

The following figures show all 946 parametric variations and the relative positions of baseline and "best" designs. Figure 3 through Figure 8, show various design space sections. Figure 9 through Figure 14 show some hull parameters and hydrostatic coefficients.

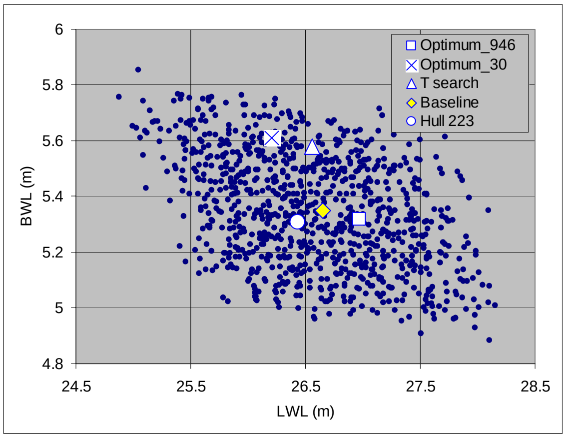

Figure 3. Design space section BWL - LWL.

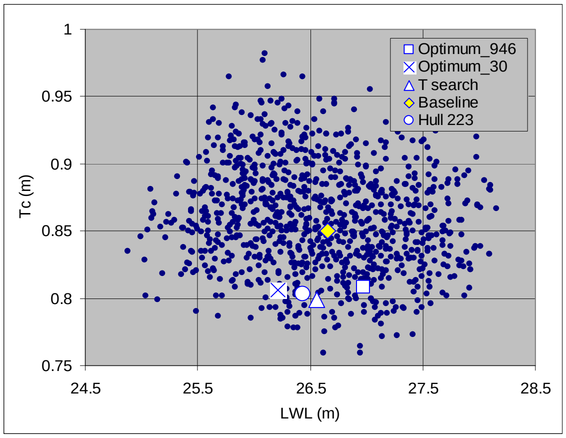

Figure 4. Design space section Tc – LWL.

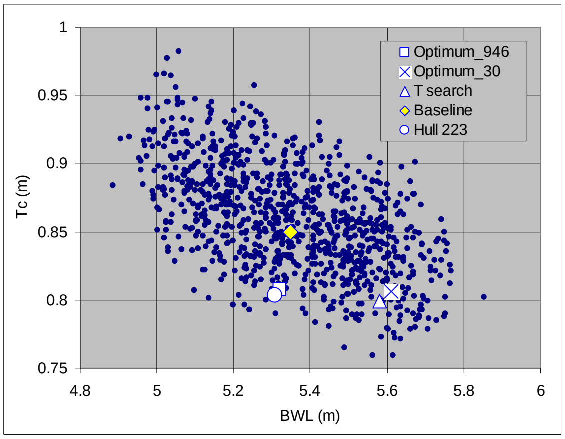

Figure 5. Design space section Tc – BWL.

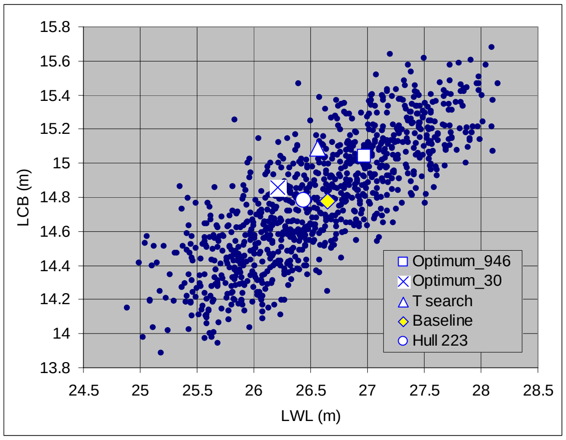

Figure 6. Design space section LCB – LWL.

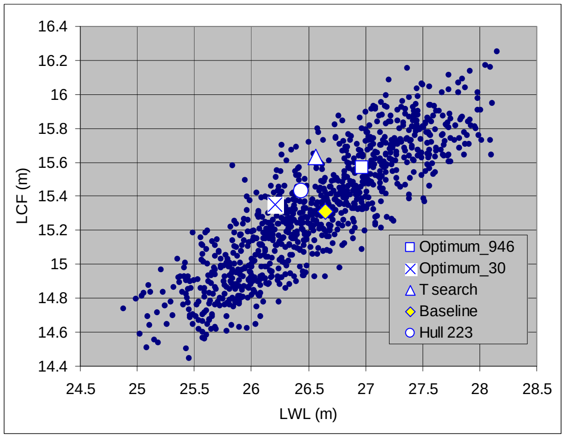

Figure 7. Design space section LCF – LWL.

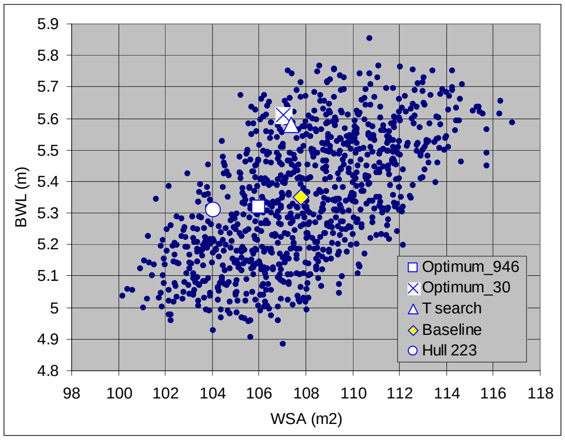

Figure 8. Design space section BWL – WSA.

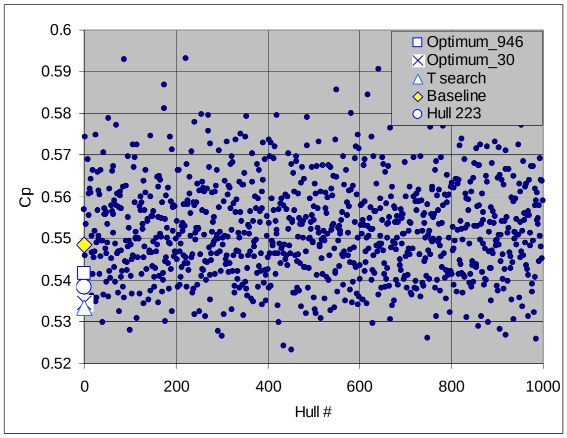

Figure 9. Cp design space.

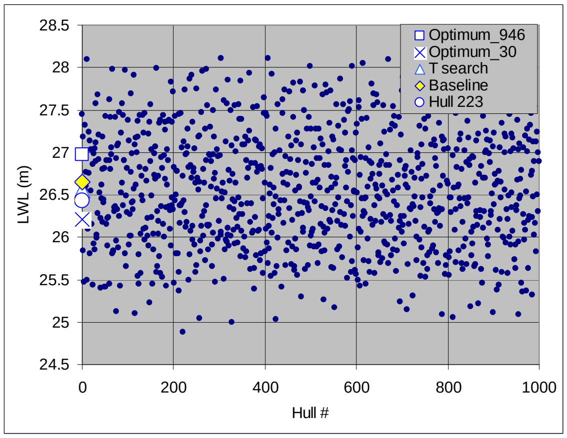

Figure 10. LWL design space.

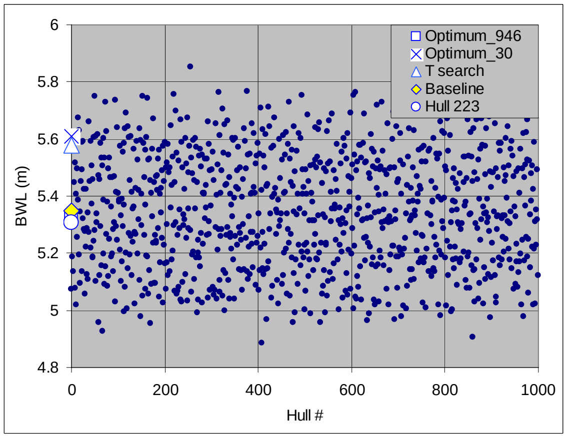

Figure 11. BWL design space.

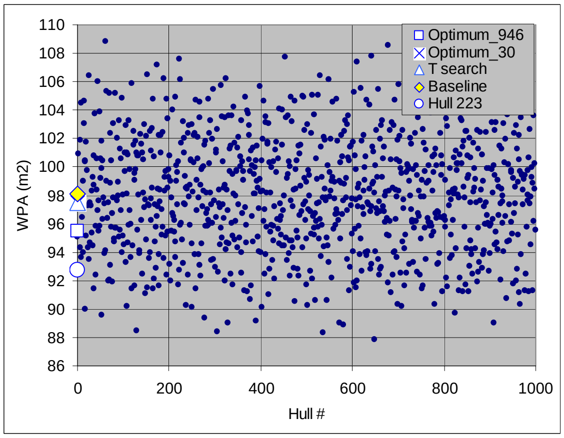

Figure 12. WPA design space.

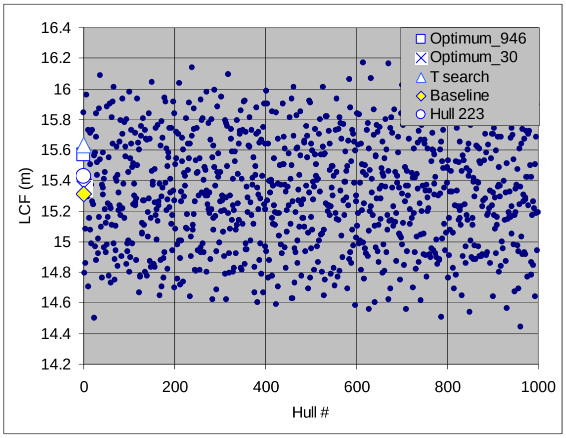

Figure 13. LCF design space.

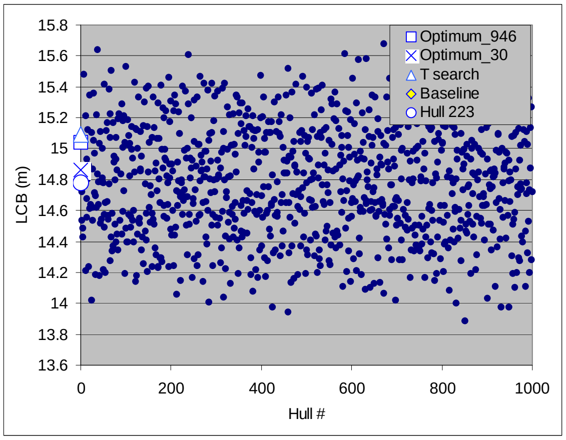

Figure 14. LCB design space.

Figure 3 through Figure 8 show that design parameters do cluster in specific regions of the design space. With the exception of BWL, all parameters show relatively low scatter, and are consistent in indicating directions for improvement. Figure 4, for example, suggests that a shallower hull draft could be beneficial as long as the LCB and LCF are increased, as shown in Figure 6 and Figure 7, respectively. The baseline's LWL seems to be located around the optimum. The "best" BWLs exhibit a relatively wide range, suggesting that perhaps the design space features a "plateau" in the 5.4 to 5.6 m range. Finally, the "best" designs indicate a trend towards lower WSA, Cp and WPA, as shown by Figure 8, Figure 9 and Figure 12, respectively.

5. CONCLUSIONS

In the quest for design optimisation, designers have devised and adopted a variety of methodologies and tools. All these fall short in achieving the fastest design for a given set of constraints. If in doubt, one just needs to look around at present and past handicap or box rule designs to see the pace of evolution. The methodology presented in this paper should not be perceived as a definite solution to the problem of performance sailboat optimisation, but as a step forward and another methodology worth considering.

Designers often face the challenge of optimising based on very many design variations for which the measures of performance would be approximate (i.e. based on regression formulas), or a relatively few variations for which those would be more accurate (i.e. based on CFD, tank testing, etc.). Given the intense and detailed work that the latter approach requires, the sampling of the design space is often coarse and the optimisation process is typically performed by trial and error.

Parametric models are essential to reduce and manage the number of free design variables for which measures of performance need to be computed. The results presented in this paper suggest that advanced parametric modelling, as performed with the FRIENDSHIP-Modeler, combined with an optimal design space sampling strategy, such as the Sobol sequence, would result in an efficient representation of the design space topology in situations where only a few design variations are available. Furthermore, the representation of the response surface (the mathematical relationship between the design parameters and measure of performance) by means of radial basis functions make possible the use of solvers for performance sailboat design optimisation, resulting in the proposed RSO methodology.

Models are not accurate and would always incur in various types of errors due to our elemental understanding of the physics of the sailing boat and the simplifications made in the models. It will always be up to the designer's talent to design a good baseline boat, define the design space to explore, select design parameter ranges, interpret results, realize trends and confirm with experience that the design development effort is on the right path.

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